Institute for Health
Metrics and Evaluation

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Raking with inequality constraints

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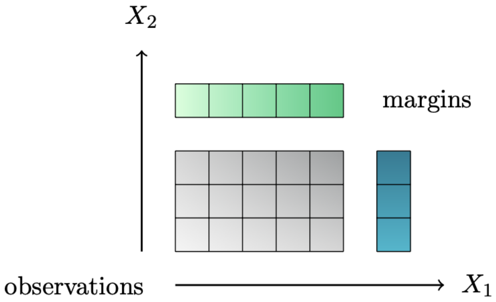
Classic raking problem



- The uncertainty on the margins is usually smaller than the uncertainty on the observations.
- The observations do not add up to the margins.
- We need to adjust the observations to match the known margins.

Classic raking problem

The raking problem can be written as an optimization problem:

$$\min_{\beta} f(\beta; y) \quad \text{s.t.} \quad A\beta = s \quad \text{with } f(\beta; y) = \sum_{i=1}^{n} f_i(\beta_i, y_i)$$

where y are the observations, s are the margins, A contains the linear constraints, and f is a strictly convex distance function.

Equivalently, we can write the dual problem:

$$\min_{\lambda} \lambda^T s + f^*(-A^T \lambda)$$

where f^* is the convex conjugate.

e.g. $f(\beta; y) = \sum_{i=1}^{n} \beta_i \log\left(\frac{\beta_i}{y_i}\right) - \beta_i + y_i$ and $f^*(z) = y^T(\exp(z) - 1)$

Prior ordinal constraints

Assume we are given prior information, in addition to the linear constraints s, in the form of inequality constraints that encode the ordering of variables

Inequality constraints

$$\min_{\beta} f(\beta, y) \quad \text{s.t.} \quad \begin{cases} A\beta = s, \\ C\beta \leq c \end{cases}$$

The feasible set may be empty.

Penalty

$$\min_{\beta} f(\beta, y) + \alpha L(c - C\beta) \quad \text{s.t.} \quad A\beta = s$$

α is a penalty parameter and L can be the logistic loss:

$$L^{\text{logit}}(x) = \sum_{i=1}^{n} \log(1 + \exp(-x_i))$$

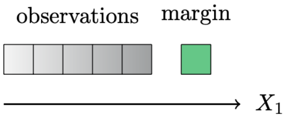
Application to infant mortality data

- Two overlapping age groups: 0-to-1-month-old and 0-to-1-year-old
- Number of deaths $y \rightarrow$ Mortality rate $m = \frac{y}{p}$ $\rightarrow$ Probability of death $q = 1 - \exp(-tm)$
- The probability of death between 0 and 1 month is lower than the probability of death between 0 and 1 year.

The raking problem becomes:

$$\min_{\beta_1, \beta_2} f(\beta_1, y_1) + f(\beta_2, y_2)$$

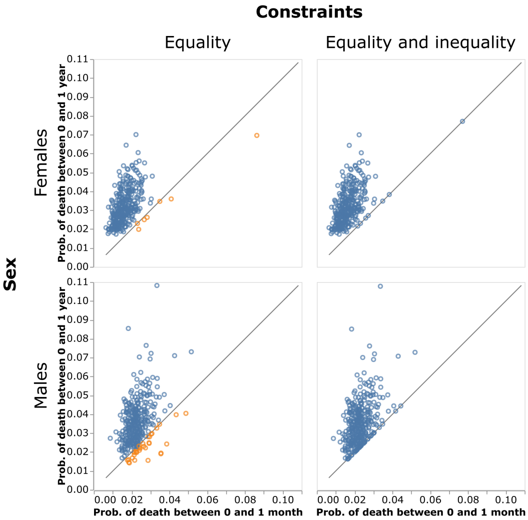
$$\text{s.t.} \quad \begin{cases} A_1 \beta_1 = s_1, \\ A_2 \beta_2 = s_2, \\ t_1 \dfrac{\beta_1}{p_1} - t_2 \dfrac{\beta_2}{p_2} \leq 0 \end{cases}$$



Comparing the two raking problems

$$\min_{\beta} f(\beta; y) \quad \text{s.t.} \quad A\beta = s$$

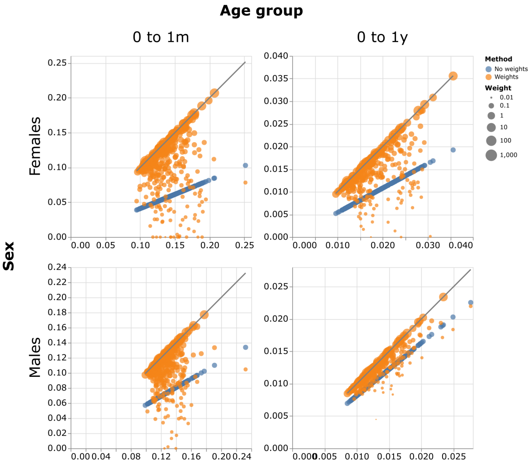
$$\min_{\beta} f(\beta, y) \quad \text{s.t.} \quad \begin{cases} A\beta = s, \\ C\beta \leq c \end{cases}$$



Adding weights to the raking problem with inequality constraints

$$f(\beta; y) = \sum_{i=1}^{n} \frac{1}{q_i} f_i(\beta_i, y_i)$$

The q_i are equal to the standard deviations of the observations y_i.



Questions?