Long-range dependence in low-frequency earthquake catalogs

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Keywords: Long-range dependence, Time series, Earthquakes

1 Introduction

Low-frequency earthquakes are small magnitude (less than 2) earthquakes, with reduced amplitudes at frequencies greater than 10 Hz relative to ordinary small earthquakes. They are often detected in subduction zones at the plate boundary between a subducting tectonic plate and the overriding plate. They are usually grouped into families of events, with all the earthquakes from a given family originating from the same small patch on the plate interface. They tend to occur in bursts, that is dozen of earthquakes are detected within a few hours or days, followed by weeks or months of quiet, with just a few earthquakes occurring. Long-range dependence is a phenomenon that may arise in the statistical analysis of time series data. It relates to the slow rate of decay of the statistical dependence between two points with increasing time interval between the points. In this paper, I study evidence of long-range dependence in low-frequency earthquake catalogs. For each family of events, the dataset contains the timing (and sometimes the magnitude) of each earthquake associated with this family. I thus translate the list of earthquake occurrence times into a discrete time series defined by the number of earthquakes per unit of time. The time series associated with two different families are correlated, but the study of this correlation is beyond the scope of this paper. In the following, I analyze each earthquake time series independently from the others. I use several graphical methods to estimate either the Hurst parameter H or the fractional differencing parameter d associated with each time series.

2 Method

Let us define a time series $X_i \left(i = 1, \cdots , N \right)$. Long-range dependence can be defined by imposing conditions on the autocorrelation $\rho_{X,\tau}$ or on the spectral density $S \left(f \right)$ (Beran, 1994).

Condition on the autocorrelation X_i is called a process with long-range dependence if there exists a real number $\alpha \in (0,1)$ and a constant c_ρ such that:

$$\lim_{\tau \to \infty} \frac{\rho_{X,\tau}}{c_\rho \tau^{-\alpha}} = 1 \tag{1}$$

Condition on the spectral density X_i is called a process with long-range dependence if there exists a real number $\alpha \in (0,1)$ and a constant c_S such that:

$$\lim_{f \to 0} \frac{S_X \left(f \right)}{c_S \left| f \right|^{-\alpha}} = 1 \tag{2}$$

Following Taqqu and Teverovsky (1998), I use graphical methods to evaluate long-range dependence. Several estimators are computed and their asymptotic behavior is used to compute either H or d. When a time series has long-range dependence, we get $0.5 < H < 1$ or $0 < d < 0.5$. We define the aggregated series:

$$X^{(m)} \left(k \right) = \frac{1}{m} \sum_{i = (k-1)m+1}^{km} X_i \text{ for } k = 1, 2, ..., \left[\frac{N}{m} \right] \tag{3}$$

The estimators used in this study are the first absolute moment:

$$AM^{(m)} = \frac{m}{N} \sum_{k=1}^{\frac{N}{m}} \left| X^{(m)} \left(k \right) - \overline{X} \right|, \tag{4}$$

the sample variance:

$$\widehat{Var} X^{(m)} = \frac{m}{N} \sum_{k=1}^{\frac{N}{m}} \left(X^{(m)} \left(k \right) - \overline{X} \right)^2, \tag{5}$$

the average $VR^{(m)}$ over k of the sample variance of the residuals:

$$\frac{1}{m} \sum_{t=1}^{m} \left(Y_k^{(m)} \left(t \right) - a - bt \right)^2 \tag{6}$$

of the linear regression of the partial sums of the time series:

$$Y_k^{(m)} \left(t \right) = \sum_{i = (k-1)m+1}^{(k-1)m+t} X_i, \tag{7}$$

the R/S statistics:

$$R/S_k^{(n)} = \frac{R_k^{(n)}}{S_k^{(n)}}, \; k = 0, \cdots , K - 1 \tag{8}$$

with:

$$R_k^{(n)} = \max_{1 \le t \le n} \left[Y_k(t) - \frac{t-1}{n} Y_k(n) \right] - \min_{1 \le t \le n} \left[Y_k(t) - \frac{t-1}{n} Y_k(n) \right], \tag{9}$$

$S_k^{(n)}$ the square root of the sample variance of $X(i)$, $i = \left[\frac{N}{K}\right] k + 1, \cdots, \left[\frac{N}{K}\right] k + n$ and:

$$Y_k(t) = \sum_{i=\left[\frac{N}{K}\right]k+1}^{\left[\frac{N}{K}\right]k+t} X_i, \tag{10}$$

and the periodogram:

$$I(f) = \frac{1}{2\pi N} \left| \sum_{t=1}^{N} X_t e^{itf} \right|^2 \text{ where } f \text{ is the frequency} \tag{11}$$

The asymptotic behavior of the five graphical estimators is given in Table 1.

Estimator	Asymptotic behavior	For
$AM^{(m)}$	m^{H-1}	Large m
$\widehat{Var}X^{(m)}$	m^{2d-1}	Large N/m and m
$VR^{(m)}$	m^{2d+1}	Large m
$R/S_k^{(n)}$	$n^{d+\frac{1}{2}}$	Large n
$I(f)$	$\|f\|^{-2d}$	$\nu \to 0$

Table 1: Asymptotic behavior of the graphical estimators.

To estimate the value of H or d, I then did a linear regression of the logarithm of the estimator over the logarithm of m, n or f. From the value of the slope, I can then get H (for the absolute moment estimator) or d (for the four other estimators).

3 Data

I use two low-frequency earthquake catalogs from the San Andreas Fault (Shelly, 2017) and Mexico (Frank et al., 2014). They are respectively fifteen and two years long, and contain 88 and 1120 time series.

4 Results

Figures 1 and 2 show the values of the Hurst parameter H and the fractional differencing parameter d for all the time series of low-frequency earthquakes for the two catalogs studied in this paper and the five methods used for the estimation. For finite variance processes, the fractional differencing parameter d and the Hurst parameter H are related by $H = d + \frac{1}{2}$. For infinite variance processes, the fractional differencing parameter d and the Hurst parameter H are related by $H = d + \frac{1}{\lambda}$ where λ is a parameter of the distribution of the innovations of a FARIMA process (e.g. the shape of the Pareto distribution).

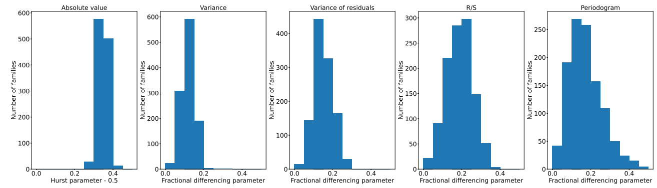

Figure 1: Distribution of the value of the Hurst parameter H or the fractional differencing parameter d for the 1120 time series from the catalog of Frank et al. (2014) for the five methods of estimation. For better comparison between the distributions of H and d, I plotted $H - 0.5$ instead of H.

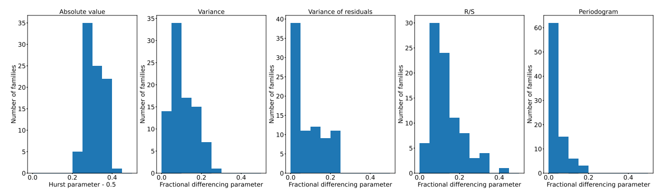

Figure 2: Same as Figure 1 for the 88 time series from the catalog of Shelly (2017).

5 Conclusion

The values of H and d obtained with two different graphical methods can be quite different from each other. However, many low-frequency earthquake time series seem to show evidence of long-range dependence, with values of the Hurst parameter between 0.5 and 1 and values of the fractional differencing parameter between 0 and 0.5. For the San Andreas Fault catalog, the values of the fractional differencing parameter d can be close to 0. An additional study is needed to establish a confidence interval for the value of d. Then, we can verify whether we can reject the hypothesis that $d = 0$, that is that there is no long-range dependence in the time series.

References

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